Core Connections Algebra
Checkpoint Materials

Notes to Students (and their Teachers)

Students master different skills at different speeds. No two students learn exactly the same way at the same time. At some point you will be expected to perform certain skills accurately. Most of the Checkpoint problems incorporate skills that you should have developed in previous courses. If you have not mastered these skills yet it does not mean that you will not be successful in this class. However, you may need to do some work outside of class to get caught up on them.

Starting in Chapter 1 and finishing in Chapter 11, there are 15 problems designed as Checkpoint problems. Each one is marked with an icon like the one above and numbered according to the chapter that it is in. After you do each of the Checkpoint problems, check your answers by referring to this section. If your answers are incorrect, you may need some extra practice to develop that skill. The practice sets are keyed to each of the Checkpoint problems in the textbook. Each has the topic clearly labeled, followed by the answers to the corresponding Checkpoint problem and then some completed examples. Next, the complete solution to the Checkpoint problem from the text is given, and there are more problems for you to practice with answers included.

Remember, looking is not the same as doing! You will never become good at any sport by just watching it, and in the same way, reading through the worked examples and understanding the steps is not the same as being able to do the problems yourself. How many of the extra practice problems do you need to try? That is really up to you. Remember that your goal is to be able to do similar problems on your own confidently and accurately. This is your responsibility. You should not expect your teacher to spend time in class going over the solutions to the Checkpoint problem sets. If you are not confident after reading the examples and trying the problems, you should get help outside of class time or talk to your teacher about working with a tutor.

Another source for help with the Checkpoint problems and other topics in *Core Connections Algebra* is the *Parent Guide with Extra Practice*. This resource is available for download free of charge at www.cpm.org.

Checkpoint Topics

1. Solving Linear Equations, Part 1 (Integer Coefficients)
2. Evaluating Expressions and the Order of Operations
3. Operations with Rational Numbers
4. Solving Linear Equations, Part 2 (Fractional Coefficients)

5A. Laws of Exponents and Scientific Notation
5B. Writing the Equation of a Line
6A. Rewriting Equations with More Than One Variable
6B. Multiplying Polynomials and Solving Equations with Parentheses
7A. Solving Problems by Writing Equations
7B. Solving Linear Systems of Equations
8. Interpreting Associations
9. Writing Exponential Equations from Situations
10A. The Exponential Web
10B. Factoring Polynomials
11. The Quadratic Web

Checkpoint 1

Problem 1-49

Solving Linear Equations, Part 1 (Integer Coefficients)

Answers to problem 1-49: a: $x = -2$, b: $x = 1\frac{1}{2}$, c: $x = 0$, d: no solution

Equations in one variable may be solved in a variety of ways. Commonly, the first step is to simplify by combining like terms. Next isolate the variable on one side and the constants on the other. Finally, divide to find the value of the variable. Note: When the process of solving an equation ends with different numbers on each side of the equal sign (for example, 2 = 4), there is *no solution* to the problem. When the result is the same expression or number on each side of the equation (for example, $x+3=x+3$) it means that *all real numbers* are solutions.

Example: Solve $4x+4x-3=6x+9$

Solution:

$4x+4x-3=6x+9$	problem
$8x-3=6x+9$	simplify
$2x=12$	add 3, subtract $6x$ on each side
$x=6$	divide

Check:
$$4(6)+4(6)-3=6(6)+9$$
$$24+24-3=36+9$$
$$48-3=45$$
$$45=45 \checkmark$$

Now we can go back and solve the original problems.

a.
$$3x+7=-x-1$$
$$4x=-8$$
$$x=-2$$

b.
$$1-2x+5=4x-3$$
$$-2x+6=4x-3$$
$$9=6x$$
$$1\tfrac{1}{2}=x$$

c.
$$4x-2+x=-2+2x$$
$$5x-2=2x-2$$
$$3x=0$$
$$x=0$$

d.
$$3x-4+1=-2x-5+5x$$
$$3x-3=3x-5$$
$$-3=-5$$
$$-3\neq-5 \Rightarrow \text{no solution}$$

Here are some more to try. Solve each equation.

1. $2x-3=-x+3$
2. $3x+2+x=x+5$
3. $6-x-3=4x-8$
4. $4x-2-2x=x-5$
5. $-x-3=2x-6$
6. $-x+2=x-5-3x$
7. $1+3x-x=x-4+2x$
8. $5x-3+2x=x+7+6x$
9. $4y-8-2y=4$
10. $-x+3=6$
11. $3y+7-y=5+2y+2$
12. $4y+7=2y+7$
13. $-x-3=2x-6$
14. $10=x+5+x$
15. $-4+3x-1=2x+1+2x$
16. $2x-7=-x-1$

Answers

1. $x=2$
2. $x=1$
3. $x=2\frac{1}{5}$
4. $x=-3$
5. $x=1$
6. $x=-7$
7. $x=5$
8. no solution
9. $y=6$
10. $x=-3$
11. all real numbers
12. $y=0$
13. $x=1$
14. $x=2\frac{1}{2}$
15 $x=-6$
16. $x=2$

Checkpoint 2

Problem 2-92

Evaluating Expressions and the Order of Operations

Answers to problem 2-92: a: –8, b: 1, c: –2, d: 17, e: –45, f: 125

In general, simplify an expression by using the Order of Operations:

- Evaluate each exponential (for example, $5^2 = 5 \cdot 5 = 25$).
- Multiply and divide each term from left to right.
- Combine like terms by adding and subtracting from left to right.

But simplify *the expressions in parentheses* or any other expressions of grouped numbers first. Numbers above or below a "fraction bar" are considered grouped. A good way to remember is to circle the terms like in the following example. Remember that terms are separated by + and – signs.

Example 1: Evaluate $2x^2 - 3x + 2$ for $x = -5$

Solution: $2(-5)^2 - 3(-5) + 2$

$2(25) - 3(-5) + 2$

$50 - (-15) + 2$

$50 + 15 + 2 = 67$

Example 2: Evaluate $5\left(\frac{x+2y}{x-y}\right)$ for $x = -3$, $y = 2$

Solution: $5\left(\frac{-3+2\cdot 2}{-3-2}\right)$

$5\left(\frac{-3+4}{-3-2}\right)$

$5\left(\frac{1}{-5}\right) = -1$

Now we can go back and solve the original problems.

a. $2x + 3y + z$

$2(-2) + 3(-3) + 5$

$-4 + -9 + 5 = -8$

b. $x - y$

$(-2) - (-3)$

$-2 + 3 = 1$

c. $2\left(\frac{x+y}{z}\right)$

$2\left(\frac{-2+-3}{5}\right)$

$2\left(\frac{-5}{5}\right) = 2(-1) = -2$

d. $3x^2 - 2x + 1$

$3(-2)^2 - 2(-2) + 1$

$3(4) - 2(-2) + 1$

$12 - (-4) + 1 = 17$

e. $3y(x + x^2 - y)$

$3(-3)(-2 + (-2)^2 - (-3))$

$3(-3)\left(-2 + 4 - (-3)\right)$

$3(-3)(5) = -45$

f. $\frac{-z^2(1-2x)}{y-x}$

$\frac{-(5)^2(1-2(-2))}{(-3)-(-2)}$

$\frac{-25(1-(-4))}{(-3)-(-2)} = \frac{-25(5)}{-1} = 125$

Here are some more to try. Evaluate each expression for $x = 4,\ y = -2,\ z = -3$.

1. $2x - 3$
2. $z^2 + 5$
3. $3z - 2y$
4. $xy - 4z$
5. $y - 2 + x$
6. $z - 8 - y$
7. $x^2 + 10x - 20$
8. $2\left(\frac{2+x}{y+1}\right)$
9. $y^2 - 3y + 7$
10. $2yz - x^2$
11. $-\frac{x}{3y}$
12. $(y + z) \cdot \frac{1}{4} x$
13. $2z(y + x^2 - x)$
14. $\frac{10+y}{3y(x+1)}$
15. $2\left(\frac{x+y}{y}\right)$
16. $(2x + y^2)(3 + z)$
17. $y - 5 + 3z^2$
18. $x^2 + 12z - 4y$
19. $\frac{2y^2 x}{x+2}$
20. $x(3 + zy) - 2x^2$
21. $z^2 + 8zy - y^2$
22. $x^3 - 4y$
23. $6z - y^2 + \frac{x+2}{z}$
24. $\frac{-y^2(xz-5y)}{3x-4y}$

Answers

1. 5
2. 14
3. –5
4. 4
5. 0
6. –9
7. 36
8. –12
9. 17
10. –4
11. $\frac{2}{3}$
12. –5
13. –60
14. $-\frac{4}{15}$
15. –2
16. 0
17. 20
18. –12
19. $5\frac{1}{3}$
20. 4
21. 53
22. 72
23. –24
24. $\frac{2}{5}$

Checkpoint 3

Problem 3-110

Operations with Rational Numbers

Answers to problem 3-110: a: $-\frac{19}{24}$, b: $4\frac{5}{6}$, c: $1\frac{2}{5}$, d: $-2\frac{2}{3}$, e: $-3\frac{7}{12}$, f: $2\frac{2}{7}$

Use the same processes with rational numbers (positive and negative fractions) as are done with integers (positive and negative whole numbers).

Example 1: Compute $\frac{1}{3}+\left(-\frac{9}{20}\right)$

Solution: When adding a positive number with a negative number, subtract the values and the number further from zero determines the sign.

$$\frac{1}{3}+-\frac{9}{20}=\frac{1}{3}\cdot\frac{20}{20}+-\frac{9}{20}\cdot\frac{3}{3}=\frac{20}{60}+-\frac{27}{60}=-\frac{7}{60}$$

Example 2: Compute $-1\frac{1}{4}-\left(-3\frac{9}{10}\right)$

Solution: Change any subtraction problem to "addition of the opposite" and then follow the addition process.

$$-1\frac{1}{4}-\left(-3\frac{9}{10}\right)\Rightarrow-1\frac{1}{4}+3\frac{9}{10}=-1\frac{5}{20}+3\frac{18}{20}=2\frac{13}{20}$$

Example 3: Compute $-1\frac{1}{4}\div7\frac{1}{2}$

Solution: With multiplication or division, if the signs are the same, then the answer is positive. If the signs are different, then the answer is negative.

$$-1\frac{1}{4}\div7\frac{1}{2}=-\frac{5}{4}\div\frac{15}{2}=-\frac{5}{4}\cdot\frac{2}{15}=-\frac{\cancel{5}\cdot\cancel{2}}{\cancel{2}\cdot2\cdot3\cdot\cancel{5}}=-\frac{1}{6}$$

Now we can go back and solve the original problems.

a. Both numbers are negative so add the values and the sign is negative.
$-\frac{2}{3}+-\frac{1}{8}=-\frac{2}{3}\cdot\frac{8}{8}+-\frac{1}{8}\cdot\frac{3}{3}=-\frac{16}{24}+-\frac{3}{24}=-\frac{19}{24}$

b. Change the subtraction to addition of the opposite.
$3\frac{1}{2}-\left(-1\frac{1}{3}\right)=3\frac{1}{2}+1\frac{1}{3}=3+\frac{1}{2}\cdot\frac{3}{3}+1+\frac{1}{3}\cdot\frac{2}{2}=3+\frac{3}{6}+1+\frac{2}{6}=4\frac{5}{6}$

c. The signs are the same so the product is positive. Multiply as usual.
$-4\frac{1}{5}\cdot-\frac{1}{3}=-\frac{21}{5}\cdot-\frac{1}{3}=\frac{7\cdot\cancel{3}\cdot1}{5\cdot\cancel{3}}=\frac{7}{5}=1\frac{2}{5}$

d. The signs are different so the quotient is negative. Divide as usual.
$-\frac{2}{3}\div\frac{1}{4}=-\frac{2}{3}\cdot\frac{4}{1}=-\frac{2\cdot4}{3\cdot1}=-\frac{8}{3}=-2\frac{2}{3}$

Solutions continue on next page. →

Solutions continued from previous page.

e. When adding a positive number with a negative number, subtract the values and the number further from zero determines the sign.
$1\frac{3}{4}+-5\frac{1}{3}=\frac{7}{4}+-\frac{16}{3}=\frac{7}{4}\cdot\frac{3}{3}+-\frac{16}{3}\cdot\frac{4}{4}=\frac{21}{12}+-\frac{64}{12}=-\frac{43}{12}=-3\frac{7}{12}$

f. The signs are the same so the quotient is positive. Divide as usual.
$-2\frac{2}{3}\div-1\frac{1}{6}=-\frac{8}{3}\div-\frac{7}{6}=-\frac{8}{3}\cdot-\frac{6}{7}=\frac{8\cdot\cancel{3}\cdot 2}{\cancel{3}\cdot 7}=\frac{16}{7}=2\frac{2}{7}$

Here are some more to try. Compute each of the following problems with rational numbers.

1. $-\frac{2}{5}+\frac{1}{2}$
2. $\frac{3}{4}-\left(-\frac{5}{12}\right)$
3. $-\frac{5}{7}+\frac{4}{5}$
4. $-1\frac{6}{7}+\left(-\frac{3}{4}\right)$
5. $\left(3\frac{1}{3}\right)\cdot\left(-\frac{2}{5}\right)$
6. $-2\frac{1}{4}\cdot\frac{2}{3}$
7. $-2\frac{7}{12}\div-\frac{1}{6}$
8. $3\frac{1}{2}+\left(-4\frac{3}{8}\right)$
9. $-1\frac{1}{4}-\left(-3\frac{1}{6}\right)$
10. $\left(2\frac{5}{9}\right)\cdot\left(-\frac{3}{7}\right)$
11. $-4\frac{3}{4}-\left(-\frac{5}{7}\right)$
12. $\frac{2}{3}\div-1\frac{4}{9}$
13. $-\frac{5}{9}\cdot1\frac{2}{3}$
14. $-\frac{3}{5}\div-1\frac{1}{10}$
15. $-5\frac{1}{2}\div-\frac{3}{4}$
16. $10\frac{5}{8}+\left(-2\frac{1}{2}\right)$
17. $5\frac{1}{5}+\left(-2\frac{2}{15}\right)$
18. $12\frac{3}{4}-\left(-1\frac{5}{8}\right)$
19. $-2\frac{7}{9}\cdot3\frac{1}{7}$
20. $-1\frac{1}{5}\div-\frac{1}{10}$
21. $5\frac{1}{12}-\left(-2\frac{6}{7}\right)$
22. $-6\frac{1}{7}\cdot-\frac{4}{5}$
23. $-1\frac{1}{8}\div2\frac{3}{4}$
24. $-2\frac{3}{5}-3\frac{1}{10}$

Answers

1. $\frac{1}{10}$
2. $1\frac{1}{6}$
3. $\frac{3}{35}$
4. $-2\frac{17}{28}$
5. $-1\frac{1}{3}$
6. $-1\frac{1}{2}$
7. $15\frac{1}{2}$
8. $-\frac{7}{8}$
9. $1\frac{11}{12}$
10. $-1\frac{2}{21}$
11. $-4\frac{1}{28}$
12. $-\frac{6}{13}$
13. $-\frac{25}{27}$
14. $\frac{6}{11}$
15. $7\frac{1}{3}$
16. $8\frac{1}{8}$
17. $3\frac{1}{15}$
18. $14\frac{3}{8}$
19. $-8\frac{46}{63}$
20. 12
21. $7\frac{79}{84}$
22. $4\frac{32}{35}$
23. $-\frac{9}{22}$
24. $-5\frac{7}{10}$

Checkpoint 4

Problem 4-86

Solving Linear Equations, Part 2 (Fractional Coefficients)

Answers to problem 4-86: a: –12, b: –24, c: $x = \frac{16}{5}$

Equations in one variable with fractional coefficients may be solved in a variety of ways. Commonly, the first step is to multiply all the terms by a common denominator to remove the fractions. Then solve in the usual way. Combine like terms. Isolate the variable on one side and the constants on the other. Finally, divide to find the value of the variable.

Example 1: Solve $\frac{1}{2}x + x - 3 = \frac{1}{3}x + 4$

Solution:

$\frac{1}{2}x + x - 3 = \frac{1}{3}x + 4$	problem
$6(\frac{1}{2}x + x - 3) = 6(\frac{1}{3}x + 4)$	multiply by the common denominator
$3x + 6x - 18 = 2x + 24$	simplify
$9x - 18 = 2x + 24$	simplify
$7x = 42$	add 18, subtract $2x$ from each side
$x = 6$	divide

Example 2: Solve for y: $\frac{y}{2} + \frac{y}{3} - 3 = y$

Solution:

$\frac{y}{2} + \frac{y}{3} - 3 = y$	problem
$6(\frac{y}{2}) + 6(\frac{y}{3}) + 6(-3) = 6(y)$	multiply by the common denominator
$3y + 2y - 18 = 6y$	simplify
$-18 = y$	subtract $5y$ from each side

Now we can go back and solve the original problems.

a.

$$\frac{1}{6}m - 3 = -5$$
$$6(\frac{1}{6}m - 3) = 6(-5)$$
$$m - 18 = -30$$
$$m = -12$$

b.

$$\frac{2}{3}x - 3 = \frac{1}{2}x - 7$$
$$(6)\frac{2}{3}x - (6)\cdot 3 = (6)\frac{1}{2}x - (6)\cdot 7$$
$$4x - 18 = 3x - 42$$
$$x = -24$$

c.

$$x + \frac{x}{2} - 4 = \frac{x}{4}$$
$$4(x + \frac{x}{2} - 4) = 4(\frac{x}{4})$$
$$4x + 2x - 16 = x$$
$$5x = 16$$
$$x = \frac{16}{5}$$

Here are some more to try. Solve each equation.

1. $\frac{2}{3}x - 7 = \frac{1}{3}x + 3$
2. $\frac{5}{4}x - 7 = x - 5$
3. $\frac{1}{3}y = \frac{1}{4}y + 7$
4. $\frac{2}{3}x + 7 = -\frac{1}{5}x + 3$
5. $\frac{x}{2} + \frac{x}{3} = 1$
6. $\frac{2x}{3} + \frac{x}{4} = 3$
7. $x + 7 + \frac{1}{3}x = \frac{3}{2}x - 2$
8. $\frac{1}{4}x + \frac{2}{3} - x = \frac{3}{5}x - \frac{1}{2}$
9. $\frac{1}{3}y + 7 + \frac{1}{6}y = \frac{1}{2}y - 1$
10. $\frac{x}{5} + \frac{2x}{3} + x = 2$
11. $3x - \frac{x}{4} + 9 = 1 - \frac{x}{2} + 8$
12. $\frac{1}{3}y - 2 + \frac{1}{2}y = 1 + y - \frac{y}{6} - 3$

Answers

1. $x = 30$
2. $x = 8$
3. $y = 84$
4. $x = -\frac{60}{13}$
5. $x = \frac{6}{5}$
6. $x = \frac{36}{11}$
7. $x = 54$
8. $x = \frac{70}{81}$
9. no solution
10. $x = \frac{15}{14}$
11. $x = 0$
12. all real numbers

Checkpoint 5A

Problem 5-55

Laws of Exponents and Scientific Notation

Answers to problem 5-55: a: 4^7, b: 1, c: $x^{-2} = \frac{1}{x^2}$, d: $\frac{y^6}{x^3}$, e: 1.28×10^4, f: 8×10^{-3}

The laws of exponents summarize several rules for simplifying expressions that have exponents. The rules are true for any base if $x \neq 0$.

$$x^a \cdot x^b = x^{(a+b)} \qquad (x^a)^b = x^{ab} \qquad \frac{x^a}{x^b} = x^{(a-b)}$$

$$x^0 = 1 \qquad x^{-a} = \frac{1}{x^a}$$

Scientific notation is a way of writing a number as a product of two factors separated by a multiplication sign. The first factor must be less than 10 and greater than or equal to 1. The second factor has a base of 10 and an integer exponent.

Example 1: Simplify $4^2 \cdot 4^{-4}$

Solution: In a multiplication problem, if the bases are the same, add the exponents and keep the base. If the answer ends with a negative exponent, take the reciprocal and change the exponent to positive.

$$4^2 \cdot 4^{-4} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

Example 2: Simplify $\frac{(x^2)^3 \cdot y^4}{x^{-2} \cdot y}$

Solution: Separate the fraction into two fractions with bases *x* and *y*. With an exponent on an exponent, multiply the exponents. Next, to divide expressions with exponents and the same base, subtract the exponents.

$$\frac{(x^2)^3 \cdot y^4}{x^{-2} \cdot y} = \frac{(x^2)^3}{x^{-2}} \cdot \frac{y^4}{y} = \frac{x^6}{x^{-2}} \cdot \frac{y^4}{y^1} = x^8 \cdot y^3 = x^8 y^3$$

Example 3: Multiply and give the answer in scientific notation. $(8 \times 10^4) \cdot (4.5 \times 10^{-2})$

Solution: Separate the number parts and the exponent parts. Multiply the number parts normally and the exponent part by adding the exponents. If this answer is not in scientific notation, change it appropriately.

$$(8 \times 10^4) \cdot (4.5 \times 10^{-2}) = (8 \times 4.5) \cdot (10^4 \times 10^{-2}) = 36 \times 10^2 = (3.6 \times 10^1) \times 10^2 = 3.6 \times 10^3$$

Now we can go back and solve the original problems.

a. $4^2 \cdot 4^5 = 4^{(2+5)} = 4^7$

b. $(5^0)^3 = 5^{0 \cdot 3} = 5^0 = 1$

c. $x^{-5} \cdot x^3 = x^{(-5+3)} = x^{-2} = \frac{1}{x^2}$

d. $(x^{-1} \cdot y^2)^3 = (\frac{1}{x} \cdot y^2)^3 = (\frac{y^2}{x})^3 = \frac{y^6}{x^3}$

e. $(8 \times 10^5) \cdot (1.6 \times 10^{-2}) = 12.8 \times 10^3 = (1.28 \times 10^1) \times 10^3 = 1.28 \times 10^4$

f. $\frac{4 \times 10^3}{5 \times 10^5} = \frac{4}{5} \times 10^{(3-5)} = 0.8 \times 10^{-2} = (8 \times 10^{-1}) \times 10^{-2} = 8 \times 10^{-3}$

Here are some more to try. Simplify each expression. For problems 19 through 24 write the final answer using scientific notation.

1. $5^4 \cdot 5^{-1}$
2. $3^3 \cdot 3^3 \cdot 3^6$
3. $x^2 \cdot (x^4)^{-2}$
4. $y^{-2} \cdot \frac{1}{y^2} \cdot y^3$
5. $3^3 \cdot 3^5 \cdot \left(\frac{1}{3}\right)^2$
6. $(3^3 \cdot 4^{-6})^2 \cdot 6^7$
7. $x^{-3} \cdot x^0$
8. $x^1 \cdot \frac{x}{x^3} \cdot \frac{y^{-2}}{y}$
9. $\frac{7^4 \cdot 9^2}{9^3 \cdot 7^2}$
10. $\frac{14^3}{14^{-2}} \cdot 14^0$
11. $\left(\frac{x^2 \cdot y^4}{x^3 \cdot y^4}\right)^0$
12. $\frac{(y^2)^3}{y^6} \cdot y^4$
13. $\left(\frac{5^2}{5^4}\right)^{-1}$
14. $(7^2 \cdot 7^3)^4$
15. $\frac{y^3 \cdot y^2 \cdot y^{-3}}{y^{-4} \cdot y^3}$
16. $\left(\frac{1}{x^4}\right)^{-2} \cdot x \cdot x^0$
17. $\left(\frac{2^4}{7^{-3}}\right)\left(\frac{7^{-2}}{2^5}\right)^{-1}$
18. $\frac{9^3 \cdot 9^{-5}}{9^0}$
19. $(4.25 \times 10^3) \cdot (2 \times 10^5)$
20. $(1.2 \times 10^4) \cdot (7.1 \times 10^{-2})$
21. $(6.9 \times 10^7) \cdot (3 \times 10^2)$
22. $(5.63 \times 10^{-6}) \cdot (4 \times 10^{-7})$
23. $(6 \times 10^{-3})^2$
24. $2.7 \times 10^4 \div 3.2 \times 10^{-2}$

Answers

1. 5^3
2. 3^{12}
3. $x^{-6} = \frac{1}{x^6}$
4. $y^{-1} = \frac{1}{y}$
5. 3^6
6. $3^{13} \cdot 2^{-17}$
7. $x^{-3} = \frac{1}{x^3}$
8. $x^{-1} \cdot y^{-3} = \frac{1}{xy^3}$
9. $\frac{7^2}{9}$
10. 14^5
11. 1
12. y^4
13. 5^2
14. 7^{20}
15. y^3
16. x^9
17. $2^9 \cdot 7^5$
18. 9^{-2}
19. 8.5×10^8
20. 8.52×10^2
21. 2.07×10^{10}
22. 2.252×10^{-12}
23. 3.6×10^{-5}
24. 8.4375×10^5

Checkpoint 5B

Problem 5-90

Writing the Equation of a Line

Answers to problem 5-90: a: $y = 2x - 3$, b: $y = -3x - 1$, c: $y = \frac{2}{3}x - 2$, d: $y \approx \frac{5}{2}x + 9$

Except for a vertical line, any line may be written in the form $y = mx + b$ where "*b*" represents the *y*-intercept of the line and "*m*" represents the slope. (Vertical lines are always of the form $x = k$.) The slope is a ratio indicating the steepness and direction of the line. The slope is calculated by $m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\Delta y}{\Delta x}$.

Example 1: Write the equation of the line with slope $-\frac{1}{2}$ and passing through the point (6, 3).

Solution: Write the general equation of a line. $y = mx + b$

Substitute the values we know for *m, x,* and *y*. $3 = -\frac{1}{2}(6) + b$

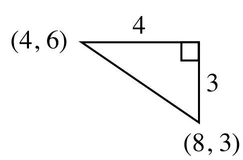
$3 = -3 + b$

Solve for *b*. $6 = b$

Write the complete equation. $y = -\frac{1}{2}x + 6$

Example 2: Write the equation of the line passing through the points (8, 3) and (4, 6).

Solution: Draw a generic slope triangle.



Calculate the slope using the given two points. $\frac{\Delta y}{\Delta x} = \frac{3}{-4}$, $m = -\frac{3}{4}$

Write the general equation of a line. $y = mx + b$

Substitute *m* and one of the points for *x* and *y*, in this case (8, 3). $3 = -\frac{3}{4}(8) + b$

$3 = -6 + b$

Solve for *b*. $9 = b$

Write the complete equation. $y = -\frac{3}{4}x + 9$

Now we can go back and solve the original problems.

a. $y = mx + b$

$17 = 2 \cdot 10 + b$

$17 = 20 + b$

$-3 = b$

$y = 2x - 3$

b. $y = mx + b$

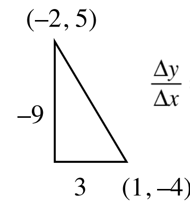
$m = -3;\ (x, y) = (-2, 5)$

$5 = -3(-2) + b$

$5 = 6 + b$

$-1 = b$

$y = -3x - 1$



$\frac{\Delta y}{\Delta x} = \frac{-9}{3} = -3$

c. Looking at the table, the entry (0, –2) tells that $b = -2$. For every increase of 2 in the *y*-value, the *x*-value increases by 3. This means the slope is $\frac{2}{3}$. If $m = \frac{2}{3}$ and $b = -2$ then the equation is $y = \frac{2}{3}x - 2$.

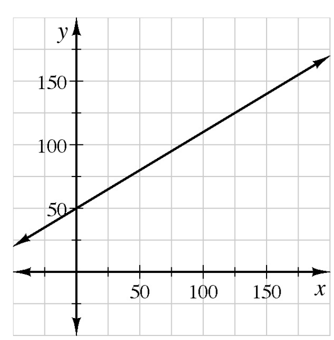
d. Looking at the graph, the *y*-intercept (*b*) is about 9. Estimating another point on the graph (2, 14) and drawing a slope triangle gives $m = \frac{\Delta y}{\Delta x} = \frac{5}{2}$. Using $y = mx + b$, the equation is $y \approx \frac{5}{2}x + 9$.

Here are some more to try. Use the given information to find an equation of the line.

1. slope = 5, through (3, 13)
2. through (1, 1), (0, 4)
3. slope = $-\frac{5}{3}$, through (3, –1)
4. through (1, 3), (–5, –15)
5. slope = –4, through (–2, 9)
6. through (2, –1), (3, –3)
7. slope = –2, through (–4, –2)
8. through (1, –4), (–2, 5)
9. slope = $\frac{1}{3}$, through (6, 9)
10. through (–4, 1), (5, –2)
11. slope = $-\frac{1}{4}$, through (–2, 6)
12. through (–3, –2), (5, –2)
13. undefined slope through (5, 2)
14. through (3, 6), (3, –1)
15.

x	–2	–1	0	1	2
y	–3	–1	1	3	5

16.



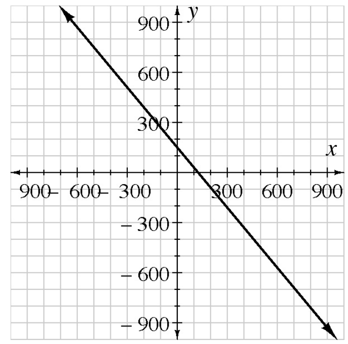
17.

x	–4	–2	0	2	4
y	1	2	3	4	5

18.

x	5	3	0	1	2
y	17	11	2	5	8

19.



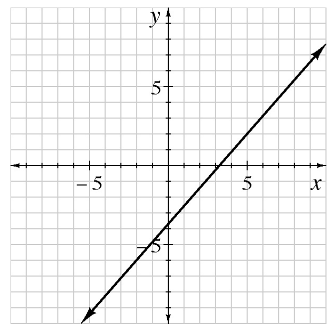
20.

x	–5	1	0	3	–2
y	7		2		4

21.

x	–1	4	2	–4	–2
y	7		–2		10

22.



Answers

1. $y = 5x - 2$
2. $y = -3x + 4$
3. $y = -\frac{5}{3}x + 4$
4. $y = 3x$
5. $y = -4x + 1$
6. $y = -2x + 3$
7. $y = -2x - 10$
8. $y = -3x - 1$
9. $y = \frac{1}{3}x + 7$
10. $y = -\frac{1}{3}x - \frac{1}{3}$
11. $y = -\frac{1}{4}x + 5\frac{1}{2}$
12. $y = -2$
13. $x = 5$
14. $x = 3$
15. $y = 2x + 1$
16. $y \approx \frac{3}{5}x + 50$
17. $y = \frac{1}{2}x + 3$
18. $y = 3x + 2$
19. $y \approx -\frac{7}{6}x + 150$
20. $y = -x + 2$
21. $y = -3x + 4$
22. $y \approx \frac{6}{5}x - 4$

Checkpoint 6A

Problem 6-29

Rewriting Equations with More Than One Variable

Answers to problem 6-29: a: $y = \frac{3x-10}{5} = \frac{3}{5}x - 2$, b: $x = \frac{y-b}{m}$, c: $r^2 = \frac{A}{\pi}$

Rewriting equations with more than one variable may be done in a variety of ways but normally you follow the same steps as you would for solving an equation with a single variable. Commonly, the first step is to multiply all the terms by a common denominator to remove all of the fractions. Then solve in the usual way. Collect like terms. Isolate the specified variable terms on one side and everything else on the other. Finally, divide or undo the exponent so that the variable is alone. The final answer will be an equation that involves variables and possibly numbers.

Example 1: Solve for *y*: $2x + 3y - 9 = 0$

Solution:

$2x + 3y - 9 = 0$	problem
$3y = -2x + 9$	subtract $2x$, add 9 on each side
$y = \frac{-2x+9}{3}$	divide by 3
$y = -\frac{2}{3}x + 3$	simplify

Example 2: Solve for *r*: $V = \frac{4}{3}\pi r^3$

Solution:

$V = \frac{4}{3}\pi r^3$	problem
$3V = 4\pi r^3$	multiply by 3 on each side
$\frac{3V}{4\pi} = r^3$	divide by 4π
$\sqrt[3]{\frac{3V}{4\pi}} = r$	cube root

Now we can go back and solve the original problems.

a.
$$-3x + 5y = -10$$
$$5y = 3x - 10$$
$$y = \frac{3x-10}{5}$$
$$y = \frac{3}{5}x - 2$$

b.
$$y = mx + b$$
$$y - b = mx$$
$$\frac{y-b}{m} = x$$

c.
$$A = \pi r^2$$
$$\frac{A}{\pi} = r^2$$

Here are some more to try. Rewrite each equation for the specified variable

1. $2x - 3y = 9$ (for x)
2. $2x - 3y = 9$ (for y)
3. $5x + 3y = 15$ (for y)
4. $4n = 3m - 1$ (for m)
5. $2w + 2l = P$ (for w)
6. $2a + b = c$ (for a)
7. $I = \frac{E}{R}$ (for R)
8. $y = \frac{1}{4}x + 1$ (for x)
9. $c^2 = a^2 + b^2$ (for b^2)
10. $V = s^3$ (for s)
11. $S = 4\pi r^2$ (for r^2)
12. $m = \frac{a+b+c}{3}$ (for a)
13. $a^3 + b = c^2$ (for a)
14. $\frac{F}{a} = m$ (for a)
15. $A = \frac{1}{2}h(b_1 + b_2)$ (for b_1)
16. $V = \frac{1}{3}\pi r^2 h$ (for h)

Answers

1. $x = \frac{3y+9}{2}$
2. $y = \frac{2x-9}{3}$
3. $y = \frac{15-5x}{3}$
4. $m = \frac{4n+1}{3}$
5. $w = \frac{P-2l}{2}$
6. $a = \frac{c-b}{2}$
7. $R = \frac{E}{I}$
8. $x = 4(y-1)$
9. $b^2 = c^2 - a^2$
10. $s = \sqrt[3]{V}$
11. $r^2 = \frac{S}{4\pi}$
12. $a = 3m - b - c$
13. $a = \sqrt[3]{c^2 - b}$
14. $a = \frac{F}{m}$
15. $b_1 = \frac{2A}{h} - b_2$
16. $h = \frac{3V}{\pi r^2}$

Checkpoint 6B

Problem 6-104

Multiplying Polynomials and Solving Equations with Parentheses

Answers to problem 6-104: a: $2x^2+6x$, b: $3x^2-7x-6$, c: 3, d: 2

Polynomials can be multiplied (changed from the area written as a product to the area written as a sum) by using the Distributive Property or generic rectangles. To solve equations with parentheses, first multiply to remove the parentheses and then solve in the usual way.

Example 1: Multiply $-5x(-2x+y)$

Solution: Using the Distributive Property $\underbrace{-5x(-2x+y)}_{\text{area as a product}}=-5x\cdot-2x+-5x\cdot y=\underbrace{10x^2-5xy}_{\text{area as a sum}}$

Example 2: Multiply $(x-3)(2x+1)$

Solution: Although the Distributive Property may be used, for this problem and other more complicated ones, it is beneficial to use a generic rectangle to find all the parts.

	$2x$	$+1$
-3		
x		

$\Rightarrow$

	$2x$	$+1$
-3	$-6x$	-3
x	$2x^2$	x

$\Rightarrow \underbrace{(x-3)(2x+1)}_{\text{area as a product}}=\underbrace{2x^2-5x-3}_{\text{area as a sum}}$

Example 3: Solve $x(2x-4)=(2x+1)(x+5)$

Solve $x(2x-4)=(2x+1)(x+5)$

Rewrite using the Distributive Property and generic rectangles.	$2x^2-4x=2x^2+11x+5$
Subtract $2x^2$	$-4x=11x+5$
Subtract $11x$	$-15x=5$
Divide by -15	$x=\frac{5}{-15}=-\frac{1}{3}$

Now we can go back and solve the original problems.

a. Using the Distributive Property: $2x(x+3) = 2x \cdot x + 2x \cdot 3 = 2x^2 + 6x$

b. Using generic rectangles:

-3		
x		
	$3x$	$+2$

$\Rightarrow$

-3	$-9x$	-6
x	$3x^2$	$2x$
	$3x$	$+2$

$\Rightarrow 3x^2 - 7x - 6$

c. $$\begin{aligned} 4y - 2(6-y) &= 6 \\ 4y - 12 + 2y &= 6 \\ 6y - 12 &= 6 \\ 6y &= 18 \\ y &= 3 \end{aligned}$$

d. $$\begin{aligned} x(2x-4) &= (2x+1)(x-2) \\ 2x^2 - 4x &= 2x^2 - 3x - 2 \\ -4x &= -3x - 2 \\ -x &= -2 \\ x &= 2 \end{aligned}$$

Here are some more to try.
Multiply in problems 1 through 15 and solve in problems 16 through 27.

1. $2x(x-1)$
2. $(3x+2)(2x+7)$
3. $(2x-1)(3x+1)$
4. $2y(x-1)$
5. $(2y-1)(3y+5)$
6. $(x+3)(x-3)$
7. $3y(x-y)$
8. $(2x-5)(x+4)$
9. $(3x+7)(3x-7)$
10. $(4x+3)^2$
11. $(x+y)(x+2)$
12. $(x-1)(x+y+1)$
13. $(2y-3)^2$
14. $(x+2)(x+y-2)$
15. $2(x+3)(3x-4)$
16. $3(c+4) = 5c+14$
17. $x - 4 = 5(x+2)$
18. $7(x+7) = 49 - x$
19. $8(x-2) = 2(2-x)$
20. $5x - 4(x-3) = 8$
21. $2x + 2(2x-4) = 244$
22. $(x-1)(x+7) = (x+1)(x-3)$
23. $(x+4)(x+3) = (x+2)(x+1)$
24. $2x - 5(x+4) = -2(x+3)$
25. $(x+2)(x+3) = x^2 + 5x + 6$
26. $(x-3)(x+5) = x^2 - 7x - 15$
27. $(x+2)(x-2) = (x+3)(x-3)$

Answers

1.	$2x^2 - 2x$	2.	$6x^2 + 25x + 14$	3.	$6x^2 - x - 1$
4.	$2xy - 2y$	5.	$6y^2 + 7y - 5$	6.	$x^2 - 9$
7.	$3xy - 3y^2$	8.	$2x^2 + 3x - 20$	9.	$9x^2 - 49$
10.	$16x^2 + 24x + 9$	11.	$x^2 + xy + 2x + 2y$	12.	$x^2 + xy - y - 1$
13.	$4y^2 - 12y + 9$	14.	$x^2 + xy + 2y - 4$	15.	$6x^2 + 10x - 24$
16.	-1	17.	$-3\frac{1}{2}$	18.	0
19.	2	20.	-4	21.	42
22.	$\frac{1}{2}$	23.	$-2\frac{1}{2}$	24.	-14
25.	all real numbers	26.	0	27.	no solution

Checkpoint 7A

Problem 7-49

Solving Problems by Writing Equations

Answers to problem 7-49: a: 13, 18, and 26 years, b: 14 months, c: \$4.80, d: 210 adult tickets and 240 student tickets.

It is common to represent situations using a single equation or a system of equations. When solving a problem involving a proportional relationship, using equal ratios is the most common method. In any case, it is important that you define the variables.

Example 1: A bag of coins are all nickels and dime. If there are 33 coins and the value is \$2.40, how many of each kind of coin are in the bag?

Solution 1: Using only one variable.

If $d =$ the number of dimes then $33 - d$ = the number of nickels.
$10d =$ the value of the dimes and $5(33 - d) =$ the value of the nickels.
The total value of the dimes and nickels together is 240 cents so:

$$10d + 5(33 - d) = 240$$

Solving yields $d = 15$ (dimes) and $33 - 15 = 18$ (nickels).

Solution 2: Using two variables.

Let $d =$ the number of dimes and $n =$ the number of nickels. Now we write two equations. The first relates the number of coins and the second relates the value of the coins.

$$d + n = 33$$
$$10d + 5n = 240$$

Solving again yields $d = 15$ (dimes) and $n = 18$ (nickels)

Example 2: Michael is swimming at a rate of 450 feet in 4 minutes. At that rate, how far will he swim in 15 minutes?

Solution: Since he is swimming at a constant rate, this is a proportional situation.

Let $d =$ the distance traveled and write two ratios comparing distance and time.

$$\frac{\text{distance}}{\text{time}} = \frac{450}{4} = \frac{d}{15}$$

Solving yields $d = 1687.5$ feet.

Now we can go back and solve the original problems.

a. Let x = the age of the youngest child so $2x$ = the age of the oldest child and $x+5=$ the age of the middle child $x+2x+x+5=57 \Rightarrow 4x=52 \Rightarrow x=13$
The youngest child is 13, the oldest $2x=26$ and the middle $x+5=18$.

b. Let w = weight in pounds and m = number of months
Katy: $w=105+2m$
James: $w=175-3m$
Since we want to know when their weights are the same we have:
$105+2m=175-3m \Rightarrow 5m=70 \Rightarrow m=14$
Their weights will be the same in 14 months.

c. Let c = the cost of the tomatoes
$\frac{\text{cost}}{\text{weight}}=\frac{\$8}{5}=\frac{c}{3} \Rightarrow 5c=24 \Rightarrow c=\$4.80$. The tomatoes will cost \$4.80.

d. Let x = the number of adult tickets
then $x+30$ = the number of student tickets
$5x+3(x+30)=1770$
Solving yields $x=210$ (adult tickets) and $x+30=240$ (student tickets).

Here are some more to try. For each problem, write one or two equations to represent the situation and then solve. Be sure to define your variable(s) and clearly answer the question.

1. A rectangle is three times as long as it is wide. The perimeter is 36 cm. Find the length of each side.

2. The sum of two consecutive odd numbers is 76. Find the numbers.

3. Nancy started the year with \$435 in the bank and is saving \$25 a week. Shane started with \$875 and is spending \$15 a week. When will they both have the same amount of money in the bank?

4. There is a \$45 tax on an \$800 scooter. How much tax would there be on a \$1000 scooter?

5. Jorge has some dimes and quarters. He has 10 more dimes than quarters and the collection of coins is worth \$2.40. How many dimes and quarters does Jorge have?

6. Tickets to the school musical are \$5.00 for adults and \$3.50 for students. If the total value of the tickets sold was \$2517.50 and 100 more students bought tickets than adults, how many adults and students bought tickets?

7. If 50 empty soda cans weigh $3\frac{1}{2}$ pounds, how much would 70 empty soda cans weigh?

8. Holmes Junior High has 125 more students than Harper Junior High. When the two schools are merged there will be 809 students. How many students currently attend each school?

9. As treasurer of her school's 4H club, Carol wants to buy gifts for all 18 members. She can buy t-shirts for $9 and sweatshirts for $15. The club has $180 to spend and Carol wants to spend all of the money. How many of each type of gift should she buy?

10. Ms. Jeffers is splitting $775 among her three sons. If the oldest gets twice as much as the youngest and the middle son gets $35 more than the youngest, how much does each boy get?

11. Evan has 356 stuffed animals, all of which are either monkeys or giraffes. The number of giraffes he owns is 17 more than twice the number of monkeys. How many monkeys does he own?

12. Oliver earns $50 per day plus $7.50 for each package he delivers. If his paycheck for the first day was $140, how many packages did he deliver that day?

13. Leticia spent $11.19 on some red and some blue candies. The bag of candies weighed 11 pounds. If the red candy costs $1.29 per pound and the blue candy costs $0.79 per pound, how much of each candy did she buy?

14. In 35 minutes, Suki's car went 25 miles. If she continues at the same rate, how long will it take her to drive 90 miles?

15. Fresh Pond has a population of 854 and is increasing by 3 people per year. Strawberry has a population of 427 and is increasing by 10 people per year. In how many years will the two villages have the same number of residents?

16. Marin County fair charges $4 to enter and $3.50 for each ride on the Screamer. Sonoma County fair charges $7 to enter but only $2 for each ride on the Screamer. How many times would you need to ride the Screamer so that you spent the same at each fair?

17. Find three consecutive numbers whose sum is 219.

18. The germination rate for zinnia seeds is 78%. This means that 78 out of every 100 seeds will sprout and grow. If James wants 60 zinnia plants for his yard, how many seeds should he plant?

Answers (Note: There are many possible correct equations.)

1. 4.5 cm and 13.5 cm
2. 37 and 39
3. 11 weeks
4. $56.25
5. 4 quarters, 14 dimes
6. 255 adults, 355 students
7. 4.9 pounds
8. Harper has 342, Holmes has 467
9. 15 t-shirts, 3 sweatshirts
10. $185, $220, $370
11. 113 giraffes, 243 monkeys
12. 12 packages
13. 5 pound of red, 6 pounds of blue
14. 126 minutes
15. 61 years
16. 2 rides
17. 72, 73, 74
18. 77 seeds

Checkpoint 7B

Problem 7-111

Solving Linear Systems of Equations

Answers to problem 7-111: a: $(-2,5)$, b: $(1,5)$, c: $(-12,14)$, d: $(2,2)$

When two equations are both in $y = mx + b$ form it is convenient to use the Equal Values Method to solve for the point of intersection. Set the two equations equal to each other to create an equation in one variable and solve for x. Then use the x-value in either equation to solve for y.

If one of the equations has a variable by itself on one side of the equation, then that expression can replace the variable in the second equation. This again creates an equation with only one variable. This is called the Substitution Method. See Example 1 below.

If both equations are in standard form (that is $ax + by = c$), then adding or subtracting the equations may eliminate one of the variables. Sometimes it is necessary to multiply before adding or subtracting so that the coefficients are the same or opposite. This is called the Elimination Method. See Example 2 below.

Sometimes the equations are not convenient for substitution or elimination. In that case one of both of the equations will need to be rearranged into a form suitable for the previously mentioned methods.

Example 1: Solve the following system.

$$4x + y = 8$$
$$x = 5 - y$$

Solution: Since x is alone in the second equation, substitute $5 - y$ in the first equation, then solve as usual.

$$4(5 - y) + y = 8$$
$$20 - 4y + y = 8$$
$$20 - 3y = 8$$
$$-3y = -12$$
$$y = 4$$

Then substitute $y = 4$ into either original equation to find x. Using the second equation $x = 1$ so the solution is $(1, 4)$.

$$x = 5 - 4$$
$$x = 1$$

Example 2: Solve the following system. $-2x + y = -7$
$3x - 4y = 8$

Solution: If we add or subtract the two equations no variable is eliminated. Notice, however, that if everything in the top equation is multiplied by 4, then when the two equations are added together, the *y*-terms are eliminated.

$$\begin{aligned} -2x + y &= -7 \\ 3x - 4y &= 8 \end{aligned} \quad\Rightarrow\quad \begin{aligned} 4(-2x + y &= -7) \\ 3x - 4y &= 8 \end{aligned} \quad\Rightarrow\quad \begin{aligned} -8x + 4y &= -28 \\ \underline{3x - 4y} &\underline{= 8} \\ -5x + 0 &= -20 \\ x &= 4 \end{aligned}$$

Using $x = 4$ into the first equation $-2(4) + y = -7 \Rightarrow -8 + y = -7 \Rightarrow y = 1$.
The solution is $(4,1)$.

Now we can go back and solve the original problems.

a. $y = 3x + 11$
$x + y = 3$

Using substitution:

$$\begin{aligned} x + (3x + 11) &= 3 \\ 4x + 11 &= 3 \\ 4x &= -8 \\ x &= -2 \\ y &= 3(-2) + 11 = 5 \end{aligned}$$

The answer is (–2, 5).

b. $y = 2x + 3$
$x - y = -4$

Using substitution:

$$\begin{aligned} x - (2x + 3) &= -4 \\ x - 2x - 3 &= -4 \\ -x - 3 &= -4 \\ -x &= -1 \\ x &= 1 \\ y &= 2(1) + 3 = 5 \end{aligned}$$

The answer is (1, 5).

c. $x + 2y = 16$
$x + y = 2$

Subtracting the second equation from the first eliminates *x*.

$$\begin{aligned} x + 2y &= 16 \\ \underline{-(x + y} &\underline{= 2)} \\ y &= 14 \\ x + 14 &= 2 \\ x &= -12 \end{aligned}$$

The answer is (–12, 14).

d. $2x + 3y = 10$
$3x - 4y = -2$

Multiplying the top by 4, the bottom by 3, and adding the equations eliminates *y*.

$$\begin{aligned} 8x + 12y &= 40 \\ \underline{9x - 12y} &\underline{= -6} \\ 17x \quad &= 34 \\ x &= 2 \\ 2(2) + 3y &= 10 \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

The answer is (2, 2).

Here are some more to try. Solve each system of equations.

1. $y = -3x$, $4x + y = 2$
2. $y = 7x - 5$, $2x + y = 13$
3. $x = -5y - 4$, $x - 4y = 23$
4. $x + y = -4$, $-x + 2y = 13$
5. $3x - y = 1$, $-2x + y = 2$
6. $2x + 5y = 1$, $2x - y = 19$
7. $x + y = 10$, $y = x - 4$
8. $y = 5 - x$, $4x + 2y = 10$
9. $3x + 5y = 23$, $y = x + 3$
10. $y - x = 4$, $2y + x = 8$
11. $2x - y = 4$, $\frac{1}{2}x + y = 1$
12. $-4x + 6y = -20$, $2x - 3y = 10$
13. $x = \frac{1}{2}y + \frac{1}{2}$, $2x + y = -1$
14. $a = 2b + 4$, $b - 2a = 16$
15. $y = 3 - 2x$, $4x + 2y = 5$
16. $6x - 2y = -16$, $4x + y = 1$
17. $4x - 4y = 14$, $2x - 4y = 8$
18. $3x + 2y = 12$, $5x - 3y = -37$
19. $x + y = 5$, $2y - x = -2$
20. $2y = 10 - x$, $3x - 2y = -2$
21. $2x - 3y = 50$, $7x + 8y = -10$
22. $3x = y - 2$, $6x + 4 = 2y$
23. $y = -\frac{2}{3}x + 4$, $y = \frac{1}{3}x - 2$
24. $5x + 2y = 9$, $2x + 3y = -3$

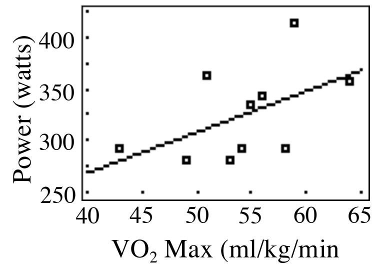
Answers

1. $(2, -6)$
2. $(2, 9)$
3. $(11, -3)$
4. $(-7, 3)$
5. $(3, 8)$
6. $(8, -3)$
7. $(7, 3)$
8. $(0, 5)$
9. $(1, 4)$
10. $(0, 4)$
11. $(2, 0)$
12. infinite solutions
13. $(0, -1)$
14. $(-12, -8)$
15. no solution
16. $(-1, 5)$
17. $(3, -\frac{1}{2})$
18. $(-2, 9)$
19. $(4, 1)$
20. $(2, 4)$
21. $(10, -10)$
22. infinite solutions
23. $(6, 0)$
24. $(3, -3)$

Checkpoint 8

Problem 8-111

Interpreting Associations



Answers to problem 8-111:
a: See graph to the right. $p = 109.62 + 3.97v$,
where p is Power (watts) and v is $V0_2$max (ml/kg/min).
b: 280 watts
c: 293 – 280 = 13 watts
d: $r = 0.51$; The linear association is positive and weak.
e: There is a weak positive linear association between power and VO_2max, with no apparent outliers. An increase of 1 ml/kg/min in VO_2max is predicted to increase power by 3.97 watts. 26.7% of the variability in the power can be explained by a linear relationship with VO_2max.

There are two reasons for modeling scattered data with a best-fit line. One is so that the trend in the data can easily be described to others without giving them the entire list of the data. The other is so that predictions can be made about points for which we do not have actual data.

When describing an association between two variables, the form, direction, strength, and outliers should be described.

The **form** (shape) can be linear, or curved, and it can contain clusters or have gaps. Residual plots are an important part of determining the appropriateness of the form of the best-fit model. However, residual plots are not included in this checkpoint.

The **direction** is positive if one variable increases as the other variable increases, and negative if one variable decreases as the other increases. If there is no apparent pattern in the scatterplot, then the variables have no association. When describing a linear association, you can use the slope, and its numerical interpretation in context, to describe the direction of the association.

The **strength** is a description of how much the data is scattered away from the line or curve of best fit. An association is "strong" if there is little scatter about the model of best fit, and "weak" if there is a lot of scatter. When describing a linear association, the correlation coefficient, r, or R-squared can be used to numerically describe and interpret the strength. See below for more about correlation coefficient and R-squared.

Outliers are data points that are far removed from the rest of the data.

A consistent best-fit line for data can be found by determining the line that makes the residuals, and hence the square of the residuals, as small as possible. We call this line the **least squares regression line** and abbreviate it LSRL. Your calculator can find the LSRL quickly.

We measure how far a prediction made by the model is from the actual observed value with a **residual**: residual = actual – predicted, where "actual" refers to the y-value of the observed data, and "predicted" refers to the y-value calculated by the LSRL when the x-value of the observed data is substituted into it. A residual has the same units as the y-axis. A residual can be graphed with a vertical segment that extends from the point to the line or curve made by the best-fit model. The length of this segment (in the units of the y-axis) is the residual. A positive residual means the predicted value is less than the actual observed value; a negative residual means the prediction is greater than the actual.

The **correlation coefficient**, r, is a measure of how much or how little data is scattered around the LSRL. It is a measure of the strength of an association that has already been determined to be linear. The correlation coefficient can take on values between –1 and 1. Correlation coefficients closer to 1 or –1 indicate that the data is less scattered around the LSRL. A positive correlation coefficient means the trend is increasing (slope is positive), while a negative correlation means the opposite. A correlation coefficient of zero means the slope of the LSRL is horizontal and there is no linear association between the variables.

***R*-squared** is found by squaring the correlation coefficient. R-squared is interpreted as follows: R^2 % of the variability in the dependent variable can be explained by a linear relationship with the independent variable.

Note: For instructions on how to use your calculator to complete these types of problems, use TI83+/84+ Calculator Instructions, which can be downloaded from www.cpm.org.

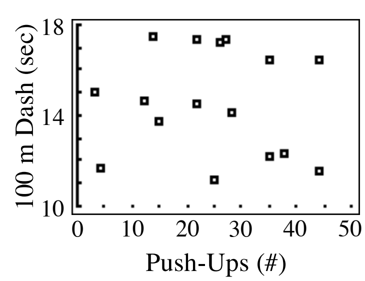
Example 1: Coach Romero is going to hold tryouts for the football team. Since timing students in the 100 m dash is time-consuming and inconvenient, he wondered if he could predict 100 m dash times from the number of push-ups a student can do. He went to the records from previous physical fitness exams and randomly chose a sample of students.

Push-Ups	100 m Dash (sec)
4	11.7
27	17.4
44	16.4
22	17.3
38	12.3
12	14.7
15	13.7
35	16.5

Push-Ups	100 m Dash (sec)
25	11.1
3	15.0
26	17.2
44	11.6
14	17.5
22	14.5
28	14.1
35	12.2
checksums: *394*	*233.2*

Use the data Coach Romero collected to describe the association between 100 m dash times and number of push-ups. Include an interpretation of the slope and R-squared. Then calculate the residual for the student that did 15 push-ups.

Solution: A statistical analysis always begins with a visual display of the data, in this case, a scatterplot. Start by entering the data into your calculator. Verify that you entered it correctly by comparing the sum of the values of each variable to the *checksum* value in the table above. Create reasonable axes for the scatterplot (often with a "windows" function).



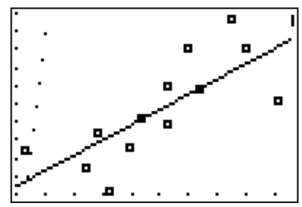
The scatterplot shows no apparent pattern. In fact, the data seems randomly scattered. There is no association between time for the 100 m dash and the number of push-ups. This means that Coach Romero cannot reasonably use push-ups to predict 100 m dash times. It is not appropriate to create a best-fit line.

Example 2: Marissa heard from her summer science camp counselor that the temperature can be determined from the number of cricket chirps. She emailed several of her relatives to ask them to stay up one night and record the temperature and the number of cricket chirps per minute. She collected the following data:

Number of Chirps per Minute	79	200	218	145	156	184	49	211	70	200	100	98	137	*checksum 1847*
Temperature (ºF)	54	71	85	65	91	77	66	81	78	90	65	74	79	*checksum 976*

Describe the association between number of cricket chirps and the temperature. Include an interpretation of the slope and *R*-squared. After Marissa collected the data and analyzed the data, she counted 107 chirps per minute on a night that the temperature was 69ºF. What is the residual for her data point?

Solution: Start by entering the data into your calculator. Verify that you entered it correctly by comparing the sum of the values of each variable to the *checksum* value in the table above. Create reasonable axes for the scatterplot (often with a "windows" function). The *x*-axis, "Number of Chirps," in the scatterplot at right goes from 40 to 230 chirps per minute, and the *y*-axis, "Temperature (ºF)," starts at 50 and goes to 100.

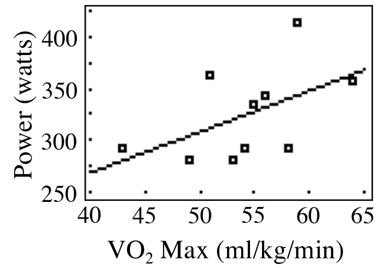


Calculating the least squares regression line is a function on the calculator. The least squares regression line for this situation is $T = 43.79 + 0.21c$, where *T* is the temperature in degrees Fahrenheit, and *c* is the number of cricket chirps per minute. The slope is 0.21, therefore an increase in one chirp per minute increases the predicted temperature by 0.21ºF. From the calculator, $r = 0.749$. You can calculate that $R^2 = (0.749)^2 = 0.56 = 56\%$, so about 56% of the variability in temperature can be explained by a linear relationship with the number of cricket chirps per minute.

Marissa observed 107 chirps. The LSRL model predicts a temperature of $43.79 + 0.21c = 43.79 + 0.21(107) = 66$ ºF. Note that Marissa's measurements for temperature were rounded to the nearest whole degree, so your prediction should also be rounded to the nearest whole degree. The actual temperature that night was 69ºF. The $\text{residual} = \text{actual} - \text{predicted} = 69 - 66 = 3°\text{F}$. Since the residual is positive, it means that the actual temperature was 3˚F more than the predicted temperature.

In conclusion, Marissa could describe the association as follows: There is a weak positive linear association between the number of cricket chirps per minute and the temperature. An increase in 1 chirp per minute increases the predicted temperature by 0.21ºF. About 56% of the variability in temperature can be explained by a linear relationship with the number of cricket chirps per minute. There are no apparent outliers.

Now we can go back and solve the original problem.



a: See graph at right. $p = 109.62 + 3.97v$, where p is power (watts) and v is VO_2max (ml/kg/min).
b: $p = 109.62 + 3.97v = 109.62 + 3.97(43) = 280$ watts. The prediction is rounded to the nearest whole number because the original measurements were rounded to the nearest whole number.
c: From the table given in the problem, the cyclist that had VO_2max of 43 ml/kg/min had a power of 293 watts.
The prediction from part (b) was 280 watts.
residual = actual – predicted = $293 - 280 = 13$ watts.
d: Using a calculator, the correlation coefficient $r = 0.51$. Since it is positive, the linear association is positive. Since the minimum possible value for r is 0, and the maximum is 1, a correlation of 0.52 is fairly weak. There is a lot of scatter in the data.
e: There is a weak positive linear association between power and VO_2max, with no apparent outliers. The slope is 3.97, which is interpreted as meaning that an increase of 1 ml/kg/min in VO_2max is predicted to increase power by 3.97 watts. $R^2 = (0.51)^2 = 0.26 = 26\%$. 26% of the variability in the power can be explained by a linear relationship with VO_2max.

Here are some more to try.

1. Describe the association in the data below, including an interpretation of the slope, r, and R-squared.

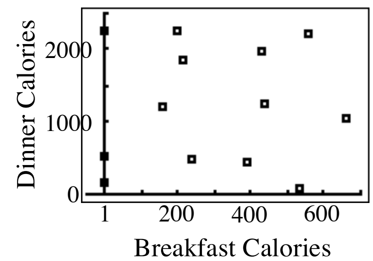
   Is school achievement associated with the amount of energy one gets from breakfast? A school psychologist collected data from some of her students. She converted each student's description of the breakfasts that they ate for one week into calories. She used a standard measure of school achievement, which has a scale from 0 to 500. Her data is below:

Breakfast Calories (kcal /week)	370	1550	840	2410	710	1970	1970	970	1560	*checksum 12350*
Achievement Score	130	350	290	400	40	450	290	170	160	*checksum 2280*

   a. Describe the association.

   b. The psychologist reports that a student who ate 1000 calories for the week at breakfast has a residual of 75 points. What was the student's achievement score?

2. Does eating a small breakfast mean that you tend to overeat at dinner on the same day? The school psychologist asked 13 students what they ate for breakfast and what they ate for dinner on the same day. She recorded the calories eaten for each meal and did an analysis. Her data follows, where d is the dinner calories, and b is the breakfast calories.



$$d = 1180 + 0.12b$$
$$r = 0.03$$

Describe the association.

3. Interpret the meaning of the correlation coefficient in the following situations:

   a. A local newspaper reports that the correlation coefficient between number of pedestrians on the sidewalk and sales tax generated by nearby stores is $r = -1.07$.

   b. The correlation coefficient between the number of hot dogs sold at the baseball game and the number of runs the Houston Homerunners baseball team scores in that game is $r = -0.94$.

   c. The correlation coefficient between the amount of sugar consumed per month and the amount of red meat consumed per month is $r = 0.10$.

4. A team of college students in a Geography class were asked to determine if there an association between latitude and mean daily temperature of cities in the United States. Latitude is measured in degrees, and indicates how far north of the equator the city is. The students gathered data from the Internet and recorded the mean January temperatures of 142 large cities and each city's latitude. The scatterplot appeared to be reasonably linear, so the students created a least squares regression line.

   The LSRL was $T = 125 - 2.3L$, where T is the mean January temperature (ºF) and L is the latitude (degrees). $r = -0.85$

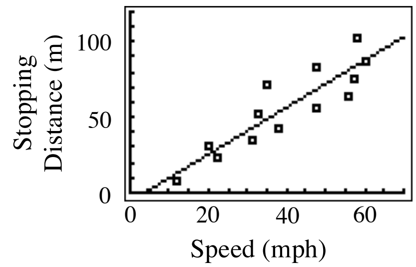
   a. Describe the association, including an interpretation of the slope and the correlation coefficient in context.

   b. A city with a latitude of 40 degrees has a residual of 35 degrees. Should the city advertise that it is a warmer than expected in winter, or that it is a winter wonderland of ice and snow because it is colder than expected in winter?

   c. What is the actual mean January temperature for the city in part (b)?

5. A consumer magazine regularly measures the stopping distance of cars when they have to brake suddenly. Their analysis, where x is the speed and y is the stopping distance, follows:

Describe the association between stopping distance and speed. Be sure to interpret the correlation coefficient.



$y = -5.62 + 1.55x$

$R^2 = 0.79$

6. The manager of a technical support call center was concerned that service was getting worse when the call center got busy. For the last 20 days, she wrote down the number of calls that day, and the average length of each call in minutes. Her data follows:

Number of Calls	20	220	142	86	281	53	306	193	145	287
Average Time per Call (min)	24	13	19	15	1	18	10	6	24	5

Number of Calls	273	152	320	262	115	218	223	92	315	223	*checksum 3926*
Average Time per Call (min)	11	12	3	5	28	3	17	23	27	9	*checksum 273*

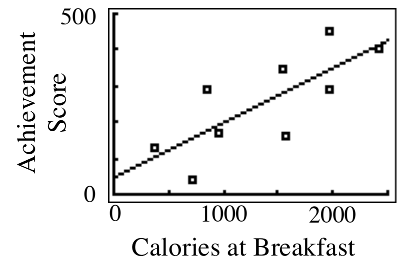
a. Does the average call length decrease as the number of calls increase? That would indicate that the quality of customer service is worse when the center is busier. Describe the association.

b. Predict the average length of a phone call on days that have 100 calls, 200 calls, and 300 calls. Use the same precision your answer as that of the measurements made by the manager.

c. What is the residual for the average time per call on a day that has 320 phone calls?

Answers

1. a. There is a moderate linear association between the number of calories eaten and school achievement, with no apparent outliers. 50% of the variability in achievement scores can be explained by the number of calories eaten at breakfast. An increase of 1 calorie is predicted to increase achievement by 0.15 points. (We note that just because there is an association, it does not imply that eating more at breakfast *causes* an increase in achievement. There may be a lurking variable that raises both breakfast calories and achievement: parent education, or socioeconomic status, or number of hours family spends together.)



   b. From the LSRL, the predicted score is 197. residual = actual – predicted; 75 = actual – 197; 272 = actual. The student's achievement score was predicted to be 270. (It appears that the achievement scores are rounded to the nearest 10 points.)

2. There is no association between the number of calories consumed at breakfast and the number of calories consumed at dinner. The correlation coefficient of nearly zero confirms that there is no association.

3. a. The newspaper made a mistake. A correlation greater than one or less than –1 is not possible.

   b. There is a strong negative linear correlation between hot dogs sold and number of runs scored. The more runs scored, the fewer hot dogs sold.

   c. There is no association between sugar and red meat consumed.

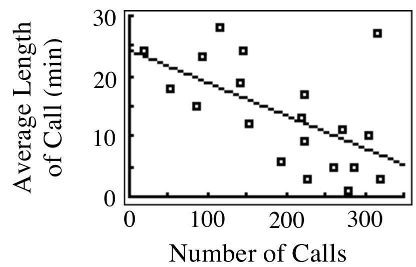
4. a. There is a moderately strong negative linear association between latitude and January temperature. (The association is negative because the slope and correlation coefficient are negative.) About 72% of the variability in January temperature can be explained by a linear relationship with latitude. An increase of 1 degree in latitude is expected to drop January temperature by 2.3 degrees.

   b. The city is 35ºF warmer than predicted by the LSRL model.

   c. The predicted January temperature for a city at 40º latitude is 33ºF. 35 = actual – 33; 68 = actual. The actual mean temperature is 68ºF.

5. There is a strong positive linear association with no outliers. 79% of the variability in stopping distance can be explained by a linear association with speed. An additional 1 mph in speed is predicted to increase the stopping distance by 1.55 meters. The correlation coefficient is 0.89. It is positive because the slope is positive. Since the value is close to one, the association is strong.

6. a. There is a moderate negative linear association. The day with 315 calls appears to be an outlier. An increase of 1 phone call per day is expected to increase the decrease length of phone calls by 0.05 minutes. 34% of the variability in the length of phone calls can be explained by a linear relationship with the number of calls per day.



b. The LSRL is $t = 24 - 0.05n$. 18, 13, 8.
The minutes have been rounded to the nearest whole number.

c. A call length of 7 minutes is predicted for a day with 320 calls.
The residual = 3 – 7 = –4. The actual length was 4 minutes less than predicted by the LSRL model.

Checkpoint 9

Problem 9-119

Writing Exponential Equations from Situations

Answers to problem 9-119: a: $y = 2.75(1.05)^{10}$, \$4.48 , b: $y = 42,000(0.75)^5$, 9967 ,
c: $60 = 25(b)^{10}$, $b = 1.09$, 9% increase

An exponential function is an equation of the form $y = ab^x$ (with $b \geq 0$).

In many cases "*a*" represents a starting or initial value, "*b*" represents the multiplier or growth/decay factor, and "*x*" represents the time. If something is increasing by a percent then the multiplier *b* is always found by adding the percent increase (as a decimal) to the number 1. If something is decreasing by a percent then the multiplier *b* is always found by subtracting the percent from the number 1.

Example 1: Movie tickets now average \$9.75 a ticket, but are increasing 15% per year. How much will they cost 5 years from now?

Solution: The equation to use is: $y = ab^x$. The initial value is $a = 9.75$. The multiplier *b* is always found by adding the percent increase (as a decimal) to the number 1 (or 100%), so $b = 1 + 0.15 = 1.15$. The time is $x = 5$. Substituting into the equation and using a calculator for the calculations: $y = ab^x = 9.75(1.15)^5 \approx 19.61$. In five years movie tickets will average about \$19.61.

Example 2: A house that was worth \$200,000 in 2005 was only worth \$150,000 in 2010. Write an equation to represent the value since 2005 and tell the percent of decrease.

Solution: The equation to use is $y = ab^x$. The initial value is $a = 200,000$, the final value $y = 150,000$, and the time $x = 5$. Substituting into the equation and solving for *b*:

$$1500000 = 2000000b^5$$
$$0.75 = b^5$$
$$b = \sqrt[5]{0.75} \approx 0.944$$

The equation is $y = 200000(0.944)^x$. $1 - 0.944 = 0.056 = 5.6\%$ decrease.

Now we can go back and solve the original problems.

a. $y = ab^x$ $\quad a = \$2.75,\ b = 1 + 0.05 = 1.05,\ x = 10$

$y = 2.75(1.05)^x \Rightarrow 2.75(1.05)^{10} \approx \$4.48$

b. $y = ab^x$ $\quad a = 42,000,\ b = 1 - 0.25 = 0.75,\ x = 5$

$y = 42,000(0.75)^x \Rightarrow 42,000(0.75)^5 \approx 9967$

c. $y = ab^x$ $\quad a = 25,\ y = 60,\ x = 10$

$60 = 25b^{10} \Rightarrow \frac{60}{25} = b^{10} \Rightarrow b = \sqrt[10]{\frac{60}{25}} \approx 1.09$ That is a 9% increase.

$y = 25(1.09)^x$

Here are some more to try.

1. A powerful computer is purchased for \$2000, but on the average loses 20% of its value each year. How much will it be worth 4 years from now?
2. If a gallon of milk costs \$3 now and the price is increasing 10% per year, how long before milk costs \$10 a gallon? (Note that guess and check will be required to solve the equation after it is written.)
3. Dinner at your grandfather's favorite restaurant now costs \$25.25 and has been increasing steadily at 4% per year. How much did it cost 35 *years ago* when he was courting your grandmother?
4. The number of bacteria present in a colony is 180 at 12 noon and the bacteria grows at a rate of 22% per hour. How many will be present at 8 p.m.?
5. A house purchased for \$226,000 has lost 4% of its value each year for the past five years. What is it worth now?
6. A 1970 comic book has appreciated 10% per year and originally sold for \$0.35. What will it be worth in 2010?
7. A Honda Accord depreciates at 15% per year. Six year ago it was purchased for \$21,000. What is it worth now?
8. Inflation is at a rate of 7% per year. Today Janelle's favorite bread costs \$3.79. What would it have cost ten years ago?
9. Ryan's motorcycle is now worth \$2500. It has decreased in value 12% each year since it was purchased. If he bought it four years ago, what did it cost new?
10. The cost of a High Definition television now averages \$1200, but the cost is decreasing about 15% per year. In how many years will the cost be under \$500?
11. A two-bedroom house in Nashville is worth \$110,000. If it appreciates at 2.5% per year, when will it be worth \$200,000?

12. Last year the principal's car was worth $28,000. Next year it will be worth $25,270. What is the annual rate of depreciation? What is the car worth now?

13. A concert has been sold out for weeks, and as the date of the concert draws closer, the price of the ticket increases. The cost of a pair of tickets was $150 yesterday and is $162 today. Assuming that the cost continues to increase at this rate exponentially:

    a. What is the daily rate of increase? What is the multiplier?

    b. What will be the cost one week from now, the day before the concert?

    c. What was the cost two weeks ago?

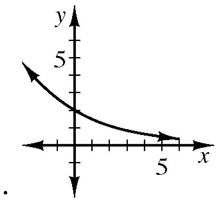
Answers

1.	$819.20	2.	≈ 13 years		
3.	$6.40 (Note that answers of $6.05 used $b = 0.96$ which is incorrect.)				
4.	≈ 883	5.	$184,274	6.	$15.84
7.	$7920	8.	$1.92	9.	$4169
10.	≈ 5 years	11.	≈ 24 years	12.	5%, $26,600
13a.	8%, 1.08	13b.	$277.64	13c.	$55.15

Checkpoint 10A

Problem 10-119

The Exponential Web



Answers to problem 10-119:
a: See graph at right.
b: $f(x) = 10(2.3)^x$
c: Anything with an initial amount of 75 and losing 15% over each time period.

An exponential function is an equation of the form $y = ab^x$ (with $b \geq 0$).

In many cases "*a*" represents a starting or initial value, "*b*" represents the multiplier and can be calculated from a table using the ratio of the *y*-values corresponding to one integer *x*-value divided by the preceding *x*-value. Also see the information in Checkpoint 9.

Example 1: Graph $y = 3 \cdot 2^x$.

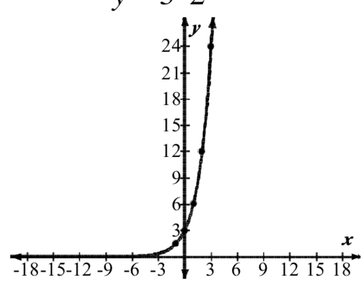
Make a table of values.

x	–1	0	1	2	3
y	1.5	3	6	12	24

This is called an increasing exponential curve.

Plot the points and connect them to form a smooth curve

$y = 3 \cdot 2^x$



Example 2: The ticket prices at African Safari Land have increased annually according to the table at right.

Write an equation that represents the cost over various years.

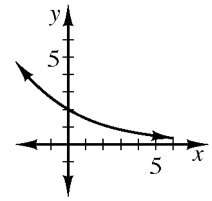
Year	Price
0	\$50
1	\$55
2	\$60.50
3	\$66.55

Solution: The initial value $a = 50$. The multiplier is the ratio of one *y*-value divided by the previous one: $b = \frac{55}{50} = \frac{60.50}{55} = \frac{66.55}{60.50} = 1.1$. The prices are increasing by 10% each year.

The equation is $y = 50(1.1)^x$.

Now we can go back and solve the original problems.

a. Make a table of values. Plot the points and connect them to form a smooth curve. See table below and graph at right.



x	–1	0	1	2	3
y	2.7	2	1.5	1.1	0.8

b. The multiplier $b=\frac{52.9}{23}=2.3$. The starting value for $x=0$ can be determined by working backwards from the value of $x=1$. $a=\frac{23}{2.3}=10$. The equation represented by the table is: $f(x)=10(2.3)^x$.

c. The initial value is 75 and the multiplier is 0.85, which means the values are decreasing at $1-0.85=0.15=15\%$. A possible situation is: A \$75 coat decreasing in value 15% each year. What will it be worth x years from now?

Here are some more to try.

Make a table of values and draw a graph of each exponential function.

1. $y=4(0.5)^x$

2. $y=2(3)^x$

3. $f(x)=5(1.2)^x$

4. $f(x)=10\left(\frac{2}{3}\right)^x$

Write an equation to represent the information in each table.

5.

x	$f(x)$
0	1600
1	2000
2	2500
3	3125

6.

x	y
1	40
2	32
3	25.6

7.

x	$f(x)$
0	1.8
1	5.76
2	18.432

8.

x	y
0	5
1	35
2	245

Write a possible context based on each equation.

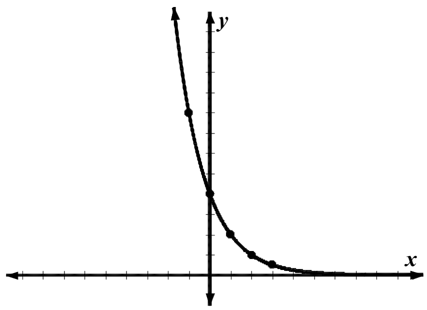
9. $y=32,500(0.85)^x$

10. $f(x)=2.75(1.025)^x$

Answers

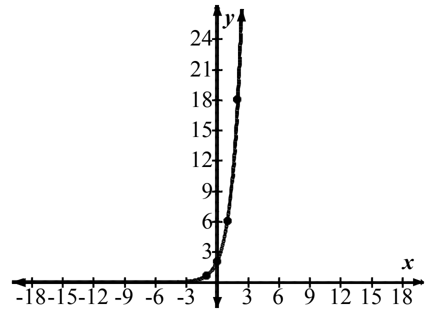
1.

x	–1	0	1	2	3
y	8	4	2	1	0.5



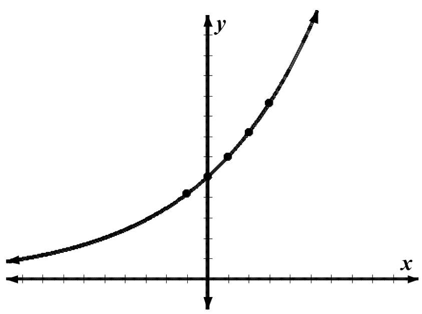
2.

x	–1	0	1	2	3
y	2/3	2	6	18	54



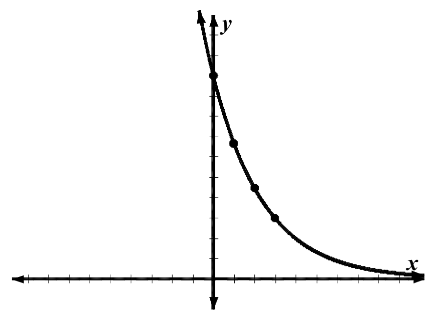
3.

x	–1	0	1	2	3
y	4.17	5	6	7.2	8.64



4.

x	–1	0	1	2	3
y	15	10	6.67	4.44	2.96



5. $1600(1.25)^x$ 6. $50(0.8)^x$ 7. $1.8(3.2)^x$ 8. $5 \cdot 7^x$

9. Possible answer: If a $32,500 car loses 15% of its value each year, what will it be worth x years from now?

10. Possible answer: A soda at the movies now costs $2.75. If the cost is increasing 2.5% per year, what will it cost x years from now?

Checkpoint 10B

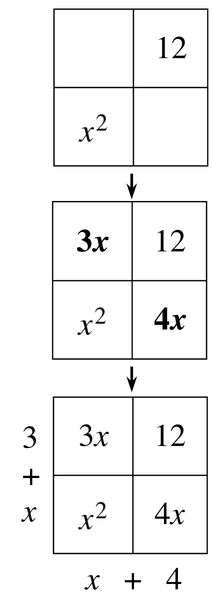
Problem 10-142

Factoring Polynomials

Answers to problem 10-142:
a: $(x-7)(x-1)$; b: $(y-5)(y+3)$; c: $7(x+3)(x-3)$; d: $(3x+4)(x+2)$

Factoring polynomials requires changing a sum into a product. It is the reverse of multiplying polynomials and using a generic rectangle is helpful.

Example 1: Factor $x^2+7x+12$.



Solution: Sketch a generic rectangle with 4 sections.

Write the x^2 and the 12 along one diagonal.

Find two terms whose product is $12 \cdot x^2 = 12x^2$ and whose sum is $7x$. That is, $3x$ and $4x$. This is the same as a Diamond Problem from Chapter 1.

Write these terms as the other diagonal.

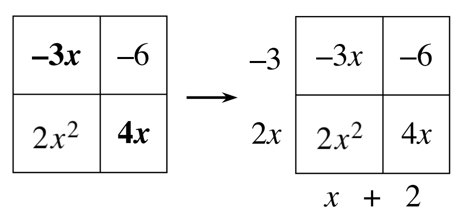
Find the base and height of the rectangle by using the partial areas.

Write the complete equation. $x^2+7x+12=(x+3)(x+4)$

Example 2: Factor $2x^2+x-6$.

Solution: Sketch a generic rectangle with 4 sections.

Write $2x^2$ and –6 along one diagonal.



Find two terms whose product is $-12x^2$ and whose sum is $1x$. That is, $4x$ and $-3x$.

Write these terms as the other diagonal.

Find the base and height of the rectangle.

Write the complete equation. $2x^2+x-6=(2x-3)(x+2)$

Now we can go back and solve the original problems.

a. $x^2 - 8x + 7$
The two terms whose product is $7x^2$ and whose sum is $-8x$ are $-1x$ and $-7x$.

$-1x$	7
x^2	**$-7x$**

$\longrightarrow$

-1	$-1x$	7
x	x^2	$-7x$
	x	$-\ 7$

$x^2 - 8x + 7 = (x-1)(x-7)$

b. $y^2 - 2y - 15$
The two terms whose product is $-15y^2$ and whose sum is $-2y$ are $-5y$ and $3y$.

$-5y$	-15
y^2	**$3y$**

$\longrightarrow$

-5	$-5y$	-15
y	y^2	$3y$
	y	$+\ 3$

$y^2 - 2y - 15 = (y-5)(y+3)$

c. $7x^2 - 63$
Although this could be shown using a generic rectangle, we recognize a common factor and the pattern of "difference of squares."

$7x^2 - 63 = 7(x^2 - 9)$ Then use the difference of squares pattern:
$a^2 - b^2 = (a+b)(a-b)$.

$7x^2 - 63 = 7(x^2 - 9) = 7(x+3)(x-3)$

d. $3x^2 + 10x + 8$
The two terms whose product is $24x^2$ and whose sum is $10x$ are $6x$ and $4x$.

$6x$	8
$3x^2$	**$4x$**

$\longrightarrow$

2	$6x$	8
$+\ x$	$3x^2$	$4x$
	$3x$	$+\ 4$

$3x^2 + 10x + 8 = (x+2)(3x+4)$

Here are some more to try. Factor each expression. Be sure to factor completely.

1. x^2+5x+6	2. $2x^2+5x+3$	3. $3x^2+4x+1$
4. $x^2-10x+25$	5. $x^2+15x+44$	6. x^2-6x-7
7. $x^2-11x+24$	8. $x^2-4x-32$	9. $4x^2+12x+9$
10. $12x^2+11x-5$	11. x^2+x-72	12. $3x^2-20x-7$
13. $x^2-11x+28$	14. $3x^2+2x-5$	15. $6x^2-x-2$
16. x^2-16	17. x^2+4x+4	18. $9x^2-16$
19. $9x^2y^2-49$	20. $a^2+8ab+16b^2$	21. $x^2y+10xy+25y$
22. $9x^4-4y^2$	23. $49x^2+1+14x$	24. $8x^2-72$
25. $2x^2+11x-6$	26. $2x^2+5x-3$	27. $x^2-3x-10$
28. $64m^2-25$	29. $c^2-10c+25$	30. $9x^2-36$
31. $4x^3-9x$	32. $4x^2-8x+4$	33. $2x^2+8$
34. $4x^2-12x+9$	35. $9x^2-18x+8$	36. $x^2-4x+16$

Answers

1. $(x+2)(x+3)$	2. $(2x+3)(x+1)$	3. $(3x+1)(x+1)$
4. $(x-5)^2$	5. $(x+11)(x+4)$	6. $(x-7)(x+1)$
7. $(x-3)(x-8)$	8. $(x-8)(x+4)$	9. $(2x+3)^2$
10. $(3x-1)(4x+5)$	11. $(x+9)(x-8)$	12. $(3x+1)(x-7)$
13. $(x-7)(x-4)$	14. $(3x+5)(x-1)$	15. $(2x+1)(3x-2)$
16. $(x+4)(x-4)$	17. $(x+2)^2$	18. $(3x+4)(3x-4)$
19. $(3xy+7)(3xy-7)$	20. $(a+4b)^2$	21. $y(x+5)^2$
22. $(3x^2+2y)(3x^2-2y)$	23. $(7x+1)^2$	24. $8(x+3)(x-3)$
25. $(x+6)(2x-1)$	26. $(x+3)(2x-1)$	27. $(x-5)(x+2)$
28. $(8m+5)(8m-5)$	29. $(c-5)^2$	30. $9(x+2)(x-2)$
31. $x(2x+3)(2x-3)$	32. $4(x-1)^2$	33. $2(x^2+4)$
34. $(2x-3)^2$	35. $(3x-4)(3x-2)$	36. not factorable

Checkpoint 11

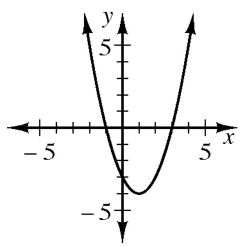
Problem 11-121

The Quadratic Web

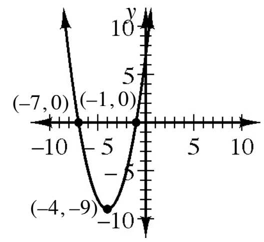
Answers to problem 11-121:

a:

x	y
–2	5
–1	0
0	–3
1	–4
2	–3
3	0
4	5



c:

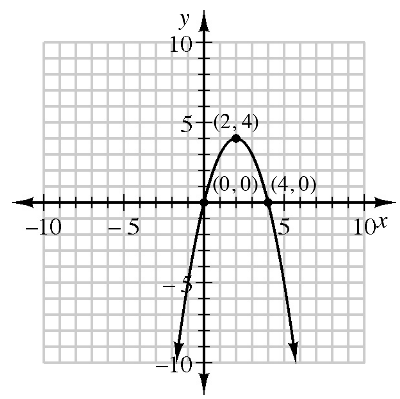


b: $y = (x-2)(x-5) = x^2 - 7x + 10$

d: 4 seconds, 256 feet

A quadratic equation is of the form $y = ax^2 + bx + c$, where $a \neq 0$. The graph is a parabola. If $a > 0$ then the parabola opens upward and the lowest point is the vertex. If $a < 0$ then the parabola opens downward and the vertex is the highest point. To find the *x*-intercepts of the parabola, set $y = 0$ and solve the resulting equation. The *x*-coordinate of the vertex is the average of the *x*-intercepts.

Example 1: Graph $y = -x^2 + 4x$.
Identify the *x*-intercepts and vertex.



Solution: Make an $x \rightarrow y$ table.

Include enough points to see a complete graph.

x	–2	–1	0	1	2	3	4	5
y	–12	–5	0	3	4	3	0	–5

Example 2: Without making a table, find the *x*-intercepts and vertex of $y = x^2 - 6x + 5$. Use that information to sketch the graph.

Solution: To find the *x*-intercepts, set $y = 0$ and solve for *x*.

$$0 = x^2 - 6x + 5$$

$$0 = (x-5)(x-1)$$

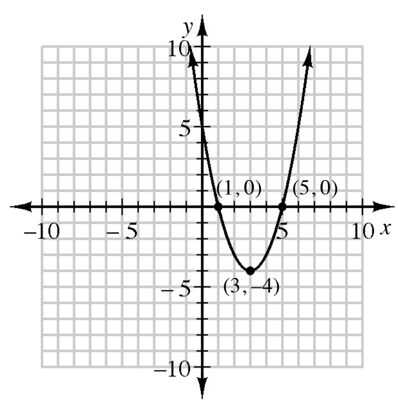
$$x = 5,\ x = 1$$

The *x*-coordinate of the vertex is $\frac{5+1}{2} = 3$.

Substitute to find the *y*-value: $y = 3^2 - 6 \cdot 3 + 5 = -4$.

The vertex is $(3, -4)$ and the *x*-intercepts are $(5, 0); (1, 0)$.

Using these points we are able to make the sketch at right.



Now we can go back and solve the original problems.

a. $y = x^2 - 2x - 3$: Make an $x \to y$ table with sufficient values to see the complete graph. See the table and graph in the answers above.

b. If $x = 2$ and $x = 5$ are intercepts then $(x - 2)$ and $(x - 5)$ are factors. The parabola opens upward so $y = (x - 2)(x - 5)$ or $y = x^2 - 7x + 10$ is a possible equation.

c. Solution: To find the *x*-intercepts, set $y = 0$ and solve for *x*.

$0 = x^2 + 8x + 7$

$0 = (x + 7)(x + 1)$

$x = -7,\ x = -1$

The *x*-coordinate of the vertex is $\frac{-7+-1}{2} = -4$.

Substitute to find the *y*-value: $y = (-4)^2 + 8(-4) + 7 = -9$.

The vertex is (–4, –9) and the *x*-intercepts are (–7, 0) and (–1, 0).

Using these points we are able to make the sketch as shown in the answers above.

d. Since the parabola opens downward, the *x*-coordinate of the vertex is the time to reach the maximum height and the *y*-coordinate of the vertex is the maximum height. Find the vertex by averaging the *x*-intercepts and substituting to find the *y*-value.

$0 = 128x - 16x^2$

$0 = 16x(8 - x)$

$x = 0,\ x = 8$

The vertex is $x = 4$; $y = 128 \cdot 4 - 16 \cdot 4^2 = 256$. After 4 seconds the rocket is 256 feet above the ground.

Here are some more to try. Find the required part of the quadratic web.

Make a table and graph.

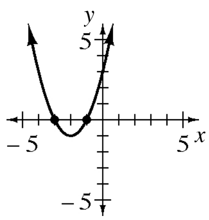
1. $y = x^2 - 4x$

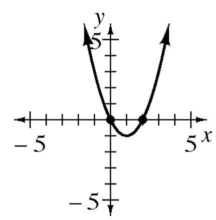
2. $y = x^2 - x - 6$

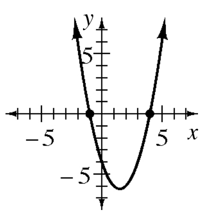
3. $y = x^2 + 4x + 1$

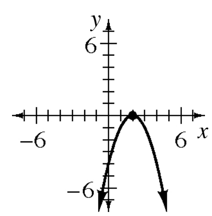
4. $y = -\frac{1}{2}x^2 - 2x + 1$

Find a possible equation for the given graph.

5. 

6. 

7. 

8. 

Use the x-intercepts and vertex to sketch a graph.

9. $y = x^2 + 6x + 8$

10. $y = x^2 + 2x$

11. $y = x^2 - 5x + 6$

12. $y = 2x^2 - 5x - 3$

Solve.

13. A path of a golf ball is given by $h = -16t^2 + 80t$ where h is the height of the ball in feet and t is the time in seconds. When does the ball hit the ground and what is the maximum height of the ball?

14. Amelia launched her science fair rocket and then backed away. The path of the rocket is given by $h = -10x^2 + 100x - 160$ where h is the height and x is the distance from Amelia. When does the rocket hit the ground and what is the maximum height of the rocket?

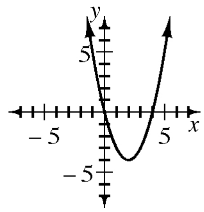
15. The revenue of a company is given by $R = -1.5x^2 + 30x$ where R is the revenue in millions and x is the number of units made in thousands. How many units should be made to maximize revenue?

16. The profit of a company is given by $P = -x^2 + 130x - 3000$. If P is the profit in millions and x is the number of items produced in millions, what number of items should be produced to maximize the profit?

Answers

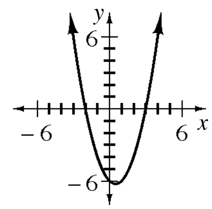
1.

x	y
–1	5
0	0
1	–3
2	–4
3	–3
4	0
5	5



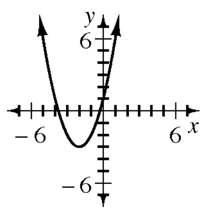
2.

x	y
–3	6
–2	0
–1	–4
0	–6
1	–6
2	–4
3	0



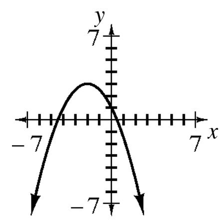
3.

x	y
–5	6
–4	1
–3	–2
–2	–3
–1	–2
0	1
1	6



4.

x	y
–5	–1.5
–4	1
–3	2.5
–2	3
–1	2.5
0	1
1	–1.5



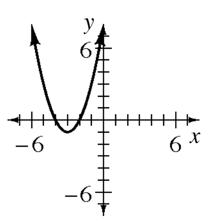
5. $y = (x+3)(x+1) = x^2 + 4x + 3$

6. $y = x(x-2) = x^2 - 2x$

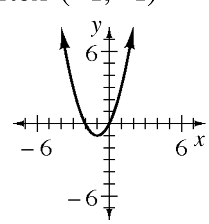
7. $y = (x+1)(x-4) = x^2 - 3x - 4$

8. $y = -(x-2)^2 = -x^2 + 4x - 4$

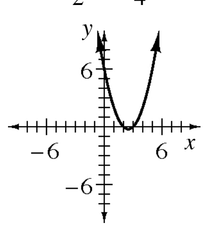
9. x-intercepts $(-4,0),(-2,0)$
vertex $(-3,-1)$



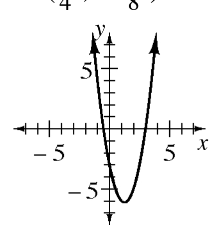
10. x-intercepts $(0,0),(-2,0)$
vertex $(-1,-1)$



11. x-intercepts $(2,0),(3,0)$
vertex $(2\frac{1}{2},-\frac{1}{4})$



12. x-intercepts $(-\frac{1}{2},0),(3,0)$
vertex $(\frac{5}{4},-\frac{49}{8})$



13. 5 seconds, 100 feet

14. 8 seconds, 90 feet

15. 10 (thousand) units

16. 65 (million) units